A leisurely stroll through the condensed world.

Logan Hyslop

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Topology, Groups, and Topological Groups

Quotient Time!

A Little Analysis

Well, who cares if it's indiscrete?

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You Could've Condensed Mathematics

Logan Hyslop

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Spaces

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Definition.

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- The rational numbers $\mathbb{Q}$ with the subspace topology.
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Every space above is Hausdorff, meaning any two points are contained in disjoint open neighborhoods, intuitively, they are far enough apart we can distinguish them topologically.

Groups

Definition.

A group G is a set together with an associative binary operation $*$, such that there is an identity element e for $*$ ($e * g = g * e = g$ for all g), and for every element $g \in G$, there is some element g^{-1} with $g^{-1} * g = g * g^{-1} = e$.

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Examples:

- $\mathbb{Z}$ under addition $+$.
- $\mathrm{GL}_n(\mathbb{R})$ under matrix multiplication.

For the remainder of this talk, we will restrict to abelian groups, meaning $a * b = b * a$ for all a and b in the group, i.e., $*$ is commutative.

Putting them Together

Definition.

A topological group is a topological space G together with a binary operation $*$ making the underlying set into a group, with the relevant group operations all being continuous. That is, such that the maps $(a, b) \mapsto a * b$ from $G \times G \to G$ and $a \mapsto a^{-1}$ from $G \to G$ are continuous.

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Examples:

- $\mathbb{R}$ under addition.
- $\mathbb{Q}$, and $\mathbb{Z}$ under addition.
- $S^1 = \{z \in \mathbb{Z} : |z| = 1\} \subseteq \mathbb{C}$ under multiplication.

Some Quotients

Whenever we have a subgroup H of an abelian group G (or more generally any continuous group homomorphism $H \to G$), we may form the quotient G/H, which sets $H = 0$, in particular giving a surjective map $\pi : G \to G/H$. If G was a topological abelian group, then this quotient inherits a topology by declaring a subset S of G/H is open if and only if the set of $x \in G$ such that $\pi(x) \in S$ is open in G.

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Example:

- $\mathbb{R}/\mathbb{Z} \simeq S^1$ by the exponential map $e^{2\pi ix}$.

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Example:

- $\mathbb{R}/\mathbb{Z} \simeq S^1$ by the exponential map $e^{2\pi ix}$.

Note that as a subspace of $\mathbb{R}$, $\mathbb{Z}$ is closed. In general, quotients by closed subgroups behave nicely.

Some More Quotients

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However, quotients don't always behave as nicely as we want them to.

Some More Quotients

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Examples:

- If we let $\mathbb{R}^{dis}$ denote the real numbers in the discrete topology (every subset is open), there is a map $\mathbb{R}^{dis} \to \mathbb{R}$ (taking a real number to itself), with quotient $\mathbb{R}/\mathbb{R}^{dis} = \{0\}$.
- $\mathbb{R}/\mathbb{Q}$ is given in the indiscrete topology (the only open subsets are the empty set and the whole space).

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- $\mathbb{R}/\mathbb{Q}$ is given in the indiscrete topology (the only open subsets are the empty set and the whole space).

In our second example, we see that the quotient is given in the indiscrete topology. This is a general phenomenon.

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More Indiscreteness

If a subgroup A of a topological abelian group B is dense in B (A has nonempty intersection with every nonempty open), then B/A is given in the indiscrete topology.

More Indiscreteness

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If a subgroup A of a topological abelian group B is dense in B (A has nonempty intersection with every nonempty open), then B/A is given in the indiscrete topology.

A couple other notable examples where this comes up are

- The space of polynomial functions with real coefficients is dense in the space of real-valued functions on $[0, 1]$.
- The space of polynomial functions with complex coefficients is dense in the space of complex analytic functions in one variable.

Definitions Galore

Definition.

A topological $\mathbb{R}$-vector space is a vector space over $\mathbb{R}$ equipped with a topology such that addition and scalar multiplication are continuous. To avoid pathologies, we usually require the topology to be sufficiently nice, in particular Hausdorff.

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Definition.

A Banach space is a topological vector space, together with a norm (some notion of size), which is complete with respect to the norm (Cauchy sequences converge).

Some Analytical Examples

- Let ℓ^0 denote the space of sequences of real numbers (x_n), all but finitely many x_n equal to 0.
- Let ℓ^1 the space of sequences (x_n) such that $\sum_{n=0}^{\infty} |x_n| < \infty$.
- Let ℓ^∞ the space of bounded sequences (x_n).

Some Analytical Examples pt. 2

- ℓ^∞ is a Banach space in the norm $\|(x_n)\| = \sup_n(|x_n|)$.
- ℓ^1 is a Banach space in the norm $\|(x_n)\| = \sum_{n=0}^{\infty} |x_n|$.
- ℓ^0 is a normed topological vector space with the norm induced by considering it as a subspace of ℓ^1, but it is not complete with respect to this norm.

Yet Another Quotient

We have $\ell^0 \subseteq \ell^1$, and this is dense in ℓ^1.

In particular, ℓ^1/ℓ^0 carries the indiscrete topology.

Similarly, we run into pathologies if we try and look at ℓ^∞/ℓ^0.

Who Cares if a Space is Indiscrete?

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When our quotients have the indiscrete topology, we "lose" the topological information.
In particular, our space is no longer a topological vector space, and fails to have some nice conditions that we like.

Who Cares if a Space is Indiscrete?

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If you are an analyst, these pathological quotients make you very upset.

A Question

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So, what can we do about it?

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If only there were some way out of this...

So what does Condensed Mean?

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Informally, we have the slogan

So what does Condensed Mean?

Informally, we have the slogan

Slogan.

While the classical story is the world from the perspective of a point, condensed math lets us view the world from the perspective of every compact Hausdorff space at once.

Formally, a condensed set is a sheaf on the site of compact Hausdorff spaces (equivalently profinite sets, extremally disconnected sets), where coverings are finite collections of jointly surjective maps.

Why would this be useful?

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Why would this be useful?

Compact Hausdorff spaces, aside from being the building blocks of many of your favorite spaces, record a lot of information.

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Compact Hausdorff spaces, aside from being the building blocks of many of your favorite spaces, record a lot of information.

There is a topology on $\mathbb{N} \cup \{\infty\}$ making it into a compact Hausdorff space, which allows it to view convergent sequences.

(Almost) a Definition

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Only a little bit more formally,

(Almost) a Definition

Only a little bit more formally,

Definition

A condensed abelian group is some magic box X, such that whenever you feed it a compact Hausdorff space S, it spits out am abelian group $X(S)$, and whenever you give it a map of compact Hausdorff spaces $f : S \to T$, it gives you back a map $f^* : X(T) \to X(S)$ which behaves nicely with respect to taking compositions, such that X satisfies a few other nice conditions, which say informally that "the global behavior of X is determined locally."

A Technical Detail

We will actually restrict usually our compact Hausdorff spaces to certain spaces called extremally disconnected sets if you have seen Stone-Cech compactifications, these give rise to all extremally disconnected sets in an appropriate sense.

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We will actually restrict usually our compact Hausdorff spaces to certain spaces called extremally disconnected sets if you have seen Stone-Cech compactifications, these give rise to all extremally disconnected sets in an appropriate sense.

What's important for us right now, is that extremally disconnected sets have good properties.

From Topological to Condensed

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There is a procedure in which every nice topological abelian group becomes a condensed abelian group.

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There is a procedure in which every nice topological abelian group becomes a condensed abelian group.

Namely, if G is a topological abelian group, then as a condensed abelian group (where we will denote it by $\underline{G}$), it is described by, for a compact Hausdorff S, $\underline{G}(S)$ is the set $\mathrm{Cont}(S, G)$ of continuous maps $r : S \to G$, where the group operation is defined by pointwise multiplication and given a map $f : S \to T$, the map $\underline{G}(T) \to \underline{G}(S)$ takes $w : T \to G$ to $w \circ f : S \to T$.

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Remark.

This suggests that for a general condensed abelian group G, we should think of $G(S)$ as "continuous maps from S to G." In fact, there is a sense in which this is literally true.

Recovery

Now that we have the idea of condensed abelian groups, we can talk about fixing our problems from earlier. Luckily, quotients of condensed abelian groups are easy to describe on the level of extremally disconnecteds.

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Namely, if you give me an extremally disconnected S, and condensed abelian groups $H \subseteq G$, then the condensed abelian group G/H has $(G/H)(S) = G(S)/H(S)$, the quotient of the ordinary abelian groups $G(S)$ by $H(S)$.

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We begin with the example $\mathbb{Q} \to \mathbb{R}$.

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Here, we have
$(\underline{\mathbb{R}}/\underline{\mathbb{Q}})(S) = \underline{\mathbb{R}}(S)/\underline{\mathbb{Q}}(S) = \operatorname{Cont}(S, \mathbb{R})/\operatorname{Cont}(S, \mathbb{Q})$.
Whereas in the classical case, $\mathbb{R}/\mathbb{Q}$ was indiscrete, and carried no topological information, there is a rich amount of information carried in the condensed abelian group $\underline{\mathbb{R}}/\underline{\mathbb{Q}}$.

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Whereas in the classical case, $\mathbb{R}/\mathbb{Q}$ was indiscrete, and carried no topological information, there is a rich amount of information carried in the condensed abelian group $\underline{\mathbb{R}}/\underline{\mathbb{Q}}$.

A similar phenomenon happens for quotients by other dense subgroups, which would otherwise be poorly behaved.

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The Other Issue?

Next, we turn to the $\mathbb{R}^{dis} \to \mathbb{R}$ example. To take the quotient here, for S extremally disconnected, we have $(\underline{\mathbb{R}}/\underline{\mathbb{R}^{dis}})(S) = \underline{\mathbb{R}}(S)/\underline{\mathbb{R}^{dis}}(S) = \mathrm{Cont}(S, \mathbb{R})/\mathrm{Cont}(S, \mathbb{R}^{dis})$.

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By the structure of extremally disconnecteds, $\mathrm{Cont}(S, \mathbb{R}^{dis})$ is the set of locally constant maps from S to $\mathbb{R}$, so that $(\underline{\mathbb{R}}/\underline{\mathbb{R}^{dis}})(S) = \{f : S \to \mathbb{R}\}/\{f : f \text{ is locally constant}\}$.

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On the other hand, $\underline{(\mathbb{R}/\mathbb{R}^{dis})}(S) = \underline{\{0\}}(S) = \mathrm{Cont}(S, \{0\}) = \{0\}$ is the zero group.

The quotient as condensed abelian groups is no longer the zero group, it now witnesses that $\mathbb{R}^{dis}$ and $\mathbb{R}$ are in fact different. In particular, $\underline{\mathbb{R}}/\underline{\mathbb{R}^{dis}} \neq \underline{\mathbb{R}/\mathbb{R}^{dis}}$.

But what about the quotients that were well-behaved?

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Fear not, quotienting out a closed subgroup in nice cases we care about recovers the usual result (as in, the condensed abelian group associated to the quotient).

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In particular, this holds in the case of locally compact abelian groups such as say, Smith spaces, which passing to filtered unions, gives the result for general complete locally convex topological $\mathbb{R}$-vector spaces.

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In particular, this holds in the case of locally compact abelian groups such as say, Smith spaces, which passing to filtered unions, gives the result for general complete locally convex topological $\mathbb{R}$-vector spaces.

Slogan

Condensed abelian groups don't destroy the things that we like, they merely add in new quotients to replace those which behaved poorly before.

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Sins of Omission

There are many points and details left out of this story for the sake of brevity, and keeping the prerequisites minimal, but the theory of condensed math, with a little more effort, gives ways to equip "measures" to rings, and discuss modules which are "complete" with respect to these measures.

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Scholze and Clausen have recently used this to develop a theory of analytic geometry, which puts nonarchimedean and archimedean geometry on level playing fields under their framework. In recent work, this theory was applied to give algebraic proofs of some results in complex analytic geometry.

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Scholze and Clausen have recently used this to develop a theory of analytic geometry, which puts nonarchimedean and archimedean geometry on level playing fields under their framework. In recent work, this theory was applied to give algebraic proofs of some results in complex analytic geometry.

Hopefully I did not do this beautiful theory an injustice.

Thank you for listening!